MATHEMAGIC ACTIVITY BOOK

CLASS - IX

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SYLLABUS GUIDELINES

CLASS - IX

Based on CBSE, ICSE & GCSE Syllabus

UNIT I: NUMBER SYSTEMS

REAL NUMBERS

Review of representation of natural numbers, integers, rational numbers on the number line. Representation of terminating/ non-terminating recurring decimals, on the number line through successive magnification. Rational numbers as recurring/terminating decimals. Examples of nonrecurring / non terminating decimals such as $\sqrt{2}, \sqrt{3}, \sqrt{5}$ etc.

Existence of non-rational numbers (irrational numbers) such as $\sqrt{2}, \sqrt{3}$, and their representation on the number line. Explaining that every real number is represented by a unique point on the number line, and conversely, every point on the number line represents a unique real number.

Existence of $\sqrt{x}$ for a given positive real number x (visual proof to be emphasized). Definition of n^{th} root of a real number.

Recall of laws of exponents with integral powers. Rational exponents with positive real bases (to be done by particular cases, allowing learner to arrive at the general laws).

Rationalization (with precise meaning) of real numbers of the type (& their combinations) $\frac{1}{a+b\sqrt{x}}$ & $\frac{1}{\sqrt{x}+\sqrt{y}}$ where x and y are natural numbers and a, b are integers.

UNIT II: ALGEBRA

1. POLYNOMIALS

Definition of a polynomial in one variable, its coefficients, with examples and counter examples, its terms, zero polynomial. Degree of a polynomial. Constant, linear, quadratic, cubic polynomials; monomials, binomials, trinomials. Factors and multiples. Zeros/roots of a polynomial/equation.

Remainder Theorem with examples and analogy to integers. Statement and proof of the Factor Theorem.

Factorization of $ax^2 + bx + c, a \neq 0$ where a, b, c are real numbers, and of cubic polynomials using the Factor Theorem.

Recall of algebraic expressions and identities. Further identities of the type

$(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx, (x \pm y)^3 = x^3 \pm y^3 \pm 2xy(x \pm y), x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$ and their use in factorization of polynomials. Simple expressions reducible to these polynomials.

2. LINEAR EQUATIONS IN TWO VARIABLES

Recall of linear equations in one variable. Introduction to the equation in two variables. Prove that a linear equation in two variables has infinitely many solutions, and justify their being written as ordered pairs of real numbers, plotting them and showing that they seem to lie on a line. Examples, problems from real life, including problems on Ratio and Proportion and with algebraic and graphical solutions being done simultaneously.

UNIT III: COORDINATE GEOMETRY

1. COORDINATE GEOMETRY

The Cartesian plane, coordinates of a point, names and terms associated with the coordinate plane, notations, plotting points in the plane, graph of linear equations as examples; focus on linear equations of the type ax + by + c = 0 by writing it as y = mx + c and linking with the chapter on linear equations in two variables.

UNIT IV: GEOMETRY

1. INTRODUCTION TO EUCLID'S GEOMETRY

History – Euclid and geometry in India. Euclid's method of formalizing observed phenomenon into rigorous mathematics with definitions, common/obvious notions, axioms/postulates, and theorems. The five postulates of Euclid.

Equivalent versions of the fifth postulate. Showing the relationship between axiom and theorem.

1. Given two distinct points, there exists one and only one line through them.
2. Two distinct lines cannot have more than one point in common.

2. LINES AND ANGLES

1. If a ray stands on a line, then the sum of the two adjacent angles so formed is 180° and the converse.
2. If two lines intersect, the vertically opposite angles are equal.
3. Results on corresponding angles, alternate angles, interior angles when a transversal intersects two parallel lines.
4. Lines, which are parallel to a given line, are parallel.
5. The sum of the angles of a triangle is 180°.
6. If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interiors opposite angles.

3. TRIANGLES

1. Two triangles are congruent if any two sides and the included angle of one triangle is equal to any two sides and the included angle of the other triangle (SAS Congruence).
2. Two triangles are congruent if any two angles and the included side of one triangle is equal to any two angles and the included side of the other triangle (ASA Congruence).
3. Two triangles are congruent if the three sides of one triangle are equal to three sides of the other triangle (SSS Congruence).
4. Two right triangles are congruent if the hypotenuse and a side of one triangle are equal (respectively) to the hypotenuse and a side of the other triangle.
5. The angles opposite to equal sides of a triangle are equal.
6. The sides opposite to equal angles of a triangle are equal.
7. Triangle inequalities and relation between 'angle and facing side' inequalities in triangles.

4. QUADRILATERALS

1. The diagonal divides a parallelogram into two congruent triangles.
2. In a parallelogram opposite sides are equal, and conversely.
3. In a parallelogram opposite angles are equal and conversely.
4. A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and equal.
5. In a parallelogram, the diagonals bisect each other and conversely.
6. In a triangle, the line segment joining the mid points of any two sides is parallel to the third side and its converse.

5. AREA

Review concept of area, recall area of a rectangle.

1. Parallelograms on the same base and between the same parallels have the same area.
2. Triangles on the same base and between the same parallels are equal in area and its converse.

6. CIRCLES

Through examples, arrive at definitions of circle related concepts, radius, circumference, diameter , chord, arc, subtended angle.

1. Equal chords of a circle subtend equal angles at the center and its converse.
2. The perpendicular from the center of a circle to a chord bisects the chord and conversely, the line drawn through the center of a circle to bisect a chord is perpendicular to the chord.
3. There is one and only one circle passing through three given non-collinear points.
4. Equal chords of a circle (or of congruent circles) are equidistant from the center(s) and conversely.
5. The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.
6. Angles in the same segment of a circle are equal.
7. If a line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, the four points lie on a circle.
8. The sum of the either pair of the opposite angles of a cyclic quadrilateral is 180^0 and its converse.

7. CONSTRUCTIONS

1. Construction of bisectors of line segments & angles, 60°, 90°, 45° angles etc, equilateral triangles.
2. Construction of a triangle given its base, sum/difference of the other two sides and one base angle.
3. Construction of a triangle of given perimeter and base angles.

UNIT V: MENSURATION

1. **AREAS**

   Area of a triangle using Hero's formula (without proof) and its application in finding the area of a quadrilateral.

2. **SURFACE AREAS AND VOLUMES**

   Surface areas and volumes of cubes, cuboids, spheres (including hemispheres) and right circular cylinders/ cones.

UNIT VI: STATISTICS & PROBABILITY

1. **STATISTICS**

   Introduction to Statistics: Collection of data, presentation of data – tabular form, ungrouped/grouped, bar graphs, histograms (with varying base lengths), frequency polygons, qualitative analysis of data to choose the correct form of presentation for the collected data. Mean, median, mode of ungrouped data.

2. **PROBABILITY**

   History, Repeated experiments and observed frequency approach to probability.

   Focus is on empirical probability. (A large amount of time to be devoted to group and to individual activities to motivate the concept; the experiments to be drawn from real - life situations, and from examples used in the chapter on statistics).

Maths Here and There

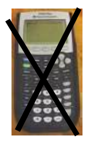
When you buy a car, follow a recipe, or decorate your home, you're using math principles. People have been using these same principles for thousands even millions of years, across countries and continents. Whether you're sailing a boat off the coast or building a house, you're using math to get things done.

How can math be so universal? First, human beings didn't invent math concepts; we discovered them. Also, the language of math is numbers, not English or German or Hindi. If we are well versed in this language of numbers, it can help us make important decisions and perform everyday tasks. Math can help us to shop wisely, buy the right insurance, remodel a home within a budget, understand population growth, or even can understand the mathematical trick behind any game.

Mathematics is the only language shared by all human beings regardless of culture, religion, or gender. Pi is still 3.14159 regardless of what country you are in. Adding up the cost of a basket full of groceries involves the same math process regardless of whether the total is expressed in dollars, rubles, or yen. With this universal language, all of us, no matter what our unit of exchange, are likely to arrive at math results the same way.

Very few people, if any, are literate in all the world's tongues—English, Chinese, Arabic, Bengali, and so on. But virtually all of us possess the ability to be "literate" in the shared language of math. This math literacy is called numeracy, and it is this shared language of numbers that connects us with people across continents and through time. It is what links ancient scholars and medieval merchants, astronauts and artists, peasants and presidents.

With this language we can explain the mysteries of the universe or the secrets of DNA. We can understand the forces of planetary motion, discover cures for catastrophic diseases, or calculate the distance from Boston to Bangkok. We can make chocolate chip cookies or save money for retirement. We can build computers and transfer information across the globe. Math is not just for calculus majors. It's for all of us. And it's not just about pondering imaginary numbers or calculating difficult equations. It's about making better daily decisions and, hopefully, leading richer, fuller lives.



Beat The Calculator

Squaring numbers in the 600s (upto 609)

1. Choose a number in the 600s (practice with smaller numbers, then progress to larger ones).
2. The first two digits of the square are 36 : 3 6 _ _ _ _
3. The next two digits will be 12 times the last 2 digits: _ _ X X _ _
4. The last two places will be the square of the last two digits: _ _ X X

Example:

If the number to be squared is **607**:

1. The first two digits are 36 : 3 6 _ _ _
2. The next two digits are 12 times the last 2 digits : 12×07= 84: _ _ 8 4 _ _
3. Square the last 2 digits: 7×7=49 :_ _ _ _ 4 9

So 607 × 607 = 368,449.

For larger numbers reverse the steps:

If the number to be squared is **625** :

1. Square the last two digits (keep carry): 25 × 25=625 (keep 6): _ _ _ _ 25
2. 12 times the last 2 digits + carry: 12 × 25 +6 (keep carry)
   = 300 + 6 = 306 : _ _ 0 6 _ _
3. 36 + carry: 36 + 3 = 39 : 39 _ _ _ _000

So 625 × 625 = 390,625.

Squaring numbers in the 700s

1. Choose a number in the 700s (practice with smaller numbers, then progress to larger ones).
2. Square the last two digits (keep the carry) : _ _ _ _X X
3. Multiply the last two digits by 14 and add the carry : _ _ X X _ _
4. The first two digits will be 49 plus the carry: X X _ _ _ _

Example:

If the number to be squared is **704** :

1. Square the last two digits : 4 × 4 = 16 : _ _ _ _1 6
2. Multiply the last two digits by 14 : 14 × 4 = 56: _ _ 5 6 _ _
3. The first two digits will be 49 : 49 _ _ _ _

Activity work sheet

4. So 704 × 704 = 495,616.

See the pattern?

If the number to be squared is **725** :

1. Square the last two digits (keep the carry): 25×25= 625: _ _ 2 5 (keep 6)
2. Multiply the last two digits by 14 and add the carry :
   14 × 25+6= 350 + 6 = 356 : _ _ 5 6 _ _ (keep 3)
3. The first two digits will be 49 plus the carry: 49 + 3 = 52 : 52 _ _ _ _
4. So 725 × 725 = 525,625.

Squaring numbers between 800 and 810

1. Choose a number between 800 and 810.
2. Square the last two digits: _ _ _ _ X X
3. Multiply the last two digits by 16 (keep the carry): _ _ X X _ _
4. Square 8, add the carry: X X _ _

Example:

If the number to be squared is **802** :

1. Square the last two digits : 2 × 2 = 4 : _ _ _ _ 0 4
2. Multiply the last two digits by 16: 16 × 2 = 32 : _ _ 3 2 _ _
3. Square 8 : 6 4 _ _ _ _
   So 802 × 802 = 643,204.

See the pattern?

If the number to be squared is **807**:

1. Square the last two digits : 7 × 7 = 49: _ _ _ _ 4 9
2. Multiply the last two digits by 16 (keep the carry): 16 × 7 =112 : _ _ 1 2 _ _
3. Square 8, add the carry (1) : 6 5 _ _ _
   So 807 × 807 = 651, 249.

Squaring numbers in the 900s

1. Choose a number in the 900s - start out easy with numbers near 1000; then go lower when expert.
2. Subtract the number from 1000 to get the difference.
3. The first three places will be the number minus the difference: X X X _ _ _.
4. The last three places will be the square of the difference: _ _ _ X X X
   (if 4 digits, add the first digit as carry).

Example:

If the number to be squared is 985 :

1. Subtract 1000 – 985 = 15 (difference)
2. Number – difference: 985 – 15 = 970 : 9 7 0 _ _ _
3. Square the difference: 15 × 15 = 225 : _ _ _ 2 2 5

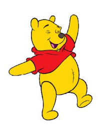
   So 985 × 985 = 970225.

See the pattern?

If the number to be squared is 920:

1. Subtract 1000 – 920 = 80 (difference)
2. Number – difference: 920 – 80 = 840 : 840 _ _ _
3. Square the difference: 80 × 80 = 6400 : _ _ _ 4 0 0
4. Carry first digit when four digits : 8 4 6 _ _ _

   So 920 × 920 = 846400.



Squaring numbers between 1000 and 1100

1. Choose a number between 1000 and 1100.
2. The first two digits are : 1,0 _ _, _ _ _
3. Find the difference between your number and 1000.
4. Multiply the difference by 2 : 1,0 X X, _ _ _
5. Square the difference : 1,0 _ _, X X X

Example:

If the number to be squared is 1007 :

1. The first two digits are : 1,0 _ _, _ _ _
2. Find the difference : 1007 – 1000 = 7
3. Two times the difference: 2 × 7 = 14 : 1,0 1 4, _ _ _
4. Square the difference: 7 × 7 = 49 : 1,0 1 4, 0 4 9

   So 1007 × 1007 = 1,014,049.

See the pattern?

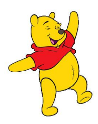
If the number to be squared is 1012:

1. The first two digits are : 1,0 _ _, _ _ _
2. Find the difference : 1012 – 1000 = 12
3. Two times the difference : 2 × 12 = 24 : 1,0 2 4, _ _ _
4. Square the difference: 12 × 12 = 144 : 1,0 2 4, 1 4 4

   So 1012 × 1012 = 1,024,144.

Activity work sheet

Start with lower numbers and then extend your expertise to all the numbers between 1000 and 1100. Remember to add the first digit as carry when the square of the difference is four digits.



Squaring numbers between 2000 and 2099

1. Choose a number between 2000 and 2099. (Start with numbers below 2025 to begin with, then graduate to larger numbers.)
2. The first two digits are: 4 0 _ _ _ _ _
3. The next two digits are 4 times the last two digits: 4 0 X X _ _ _
4. For the last three digits, square the last two digits in the number chosen (insert zeros when needed) : 4 0 _ _ X X X

Example:

If the number to be squared is **2003**:

1. The first two digits are: 4 0 _ _ _ _ _
2. The next two digits are 4 times the last two: 4 × 3 =12 : _ _ 1 2 _ _ _
3. For the last three digits, square the last two : 3 × 3 =9 : _ _ _ _ 0 0 9

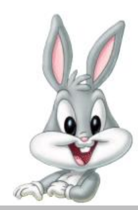
   So 2003 × 2003 =4,012,009.

See the pattern?

For larger numbers, reverse the order:

1. If the number to be squared is **2025**:
2. For the last three digits, square the last two:25×25=625: _ _ _ _ 6 2 5
3. The middle two digits are 4 times the last two (keep the carry):
   4 × 25 = 100 (keep carry of 1): _ _ 0 0 _ _ _
4. The first two digits are 40 + the carry: 40 + 1 = 41 : 4 1 _ _ _ _ _

   So 2025 × 2025 = 4,100,625.



Now its your chance

Square the following numbers.

1. 631 **2.** 605 **3.** 650 **4.** 675 **5.** 685

Square the following numbers.

1. 710 **2.** 724 **3.** 734 **4.** 770 **5.** 790

Square the following numbers.

1. 801 **2.** 805 **3.** 802 **4.** 804 **5.** 808

Square the following numbers.

1. 915 **2.** 927 **3.** 1015 **4.** 2045 **5.** 2066

Peoples in Mathematics

Thales

Thales (640 - 546 B.C.) was a philosopher, who studied mathematics, astronomy, physics and other sciences. He was born in Greece but went to Egypt to study. He measured the height of a pyramid using the idea of similarity and predicted the date of an eclipse of the sun. He is sometimes called the father of mathematics and astronomy.

Pythagoras

Pythagoras (580- 500 B.C.) was a philosopher, who studied mathematics, music and other subjects. He is famous because of the theorem named him.

William Jones

Willam jones gave the symbol of π to denote the ratio of the circumference to the diameter of a circle.

Euclid

Euclid, the Greek mathematician who lived in Alexandria is famous for geometry. He wrote 13 volumes on geometry, which became the most important works in the study of geometry and have been used throughout the world. He was born in 300 BC in Greece.

Archimedes

Archimedes (287-212 B.C.) was a famous Greek physicist and mathematician. He made important discoveries in geometry, hydrostatics and mechanics. He also invented the principle of lever and gave the concept of density.

Leonardo da Vinci

He used perspective to paint solid figures on a plane canvas. Perspective is a way of showing three dimensional objects on paper. His enquiring scientific mind led him to investigate every aspect of natural world from anatomy to aerodynamics.

Nicolaus Copernicus

Copernicus (1473-1543) was a famous astronomer, mathematician and physicist of Poland. His famous idea was that the sun is the centre of the universe and the earth revolves round the sun and